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Holographic space-time from the Big Bang to the de Sitter era

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Abstract
I review the holographic theory of space-time and its applications to cosmology. Much of this has appeared before, but this discussion is more unified and concise. I also include some material on work in progress, whose aim is to understand compactification in terms of finite-dimensional super-algebras. This is an expanded version of a lecture I gave at the conference on Liouville Quantum Gravity and Statistical Systems, in memory of Alexei Zamolodchikov, at the Poncelet Institute in Moscow, 21–24 June 2008.

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1. Introduction

1.1. For Alyosha

This paper was first prepared as a lecture to be delivered at the conference honoring the memory of the great mathematical physicist Alexei Zamolodchikov. Alyosha's untimely death was a shock and a tragedy for many of us around the world, but especially for my good friend, Alyosha's brother, Sasha. I was honored to be invited to speak at this conference, and I will remember it for a long time. Alyosha was a great physicist and a great man, and his friends gave him the only kind of send off such a man deserves: a celebration of his science and his life.

1.2. Holographic space-time

The holographic theory of space-time is an attempt to construct a general theory of quantum gravity, which will include known string theory models as special cases. It is more flexible and local than the existing formulation of string theory, and its general principles are *background* *independent*[1]. However, the holographic formalism immediately reveals the different nature of the variables for space-times with different asymptotics. The fact that the dynamical formulation of string theory depends on the space-time asymptotics has been an uncomfortable, but nonetheless valid conclusion that many string theorists have drawn from existing models.

A second advantage of the holographic formalism (in this author's eyes at least) is that it makes an immediate connection between supersymmetry and the structure of space-time. Indeed, the fundamental quantum variables, which are interpreted geometrically as the orientations of pixels on a holographic screen, can also be viewed as the degrees of freedom of supersymmetric particles penetrating the screen. More precisely, they become supersymmetric particle variables in the limit of large area screens. In this formalism, it is impossible to make a Poincaré invariant theory, which does not contain the degenerate superpartners of all particles in the theory.

The holographic formalism also sheds some light on the question of *moduli stabilization*, which has haunted much of the history of string theory. In this formalism, a local description[2] of any compact manifold always assigns a finite dimension to its algebra of continuous functions[3]. The most attractive way to do this for even dimensional manifolds is to assume that they have a symplectic structure and construct the geometry by geometric quantization, thus assigning them a finite-dimensional non-commutative algebra of functions. We will see that this kind of fuzzy compactification is also an appropriate way to think about the geometry of the holographic screen of a finite causal diamond in the non-compact dimensions. From this point of view then, moduli take on a sequence of discrete values, and continuous moduli can result only from infinite limits in space-times which admit causal diamonds of an arbitrarily large area. In particular, the hypothesis [5], which we will review below, that the quantum theory of de Sitter space has a finite-dimensional Hilbert space already implies that compact extra dimensions will have stabilized moduli, if space-time is asymptotically de Sitter in the future.

2. Holographic space-time

The basic building block of holographic space-time is not a point, but a quantum causal diamond. This is the quantum gravity construct to which a geometrical causal diamond is a classical approximation. A geometrical causal diamond is the intersection of the interior of the forward light-cone of a point P, with that of the backward light-cone of a point Q in the causal future of P, in some Lorentzian space-time. The holographic screen of such a diamond is the maximal area space-like $d-2$ surface on its boundary. The covariant entropy bound [2] assigns a finite entropy to a diamond with a finite area screen. Fischler and I [4] interpreted this as the entropy of the maximally uncertain density matrix, the logarithm of the dimension of the diamond's Hilbert space.

A pixel on the holographic screen is an element of a basis in the associative algebra of functions on the screen. We will assign a finite-dimensional Hilbert space to each pixel, and therefore a finite area screen must correspond to an algebra with a finite basis. We will allow

[1] Actually this phrase is somewhat misleading. It implicitly views quantum gravity as some sort of path integral over geometries, with different backgrounds arising as stationary points of the integral. As we will see, this is completely wrong in the holographic theory. Geometry arises instead as a collective variable of a system whose fundamental formulation does not involve summing over geometries. In particular, the quantum analogs of the causal structure and conformal factor of a given holographic model are completely fixed by its kinematics. The quantum variables are orientations of pixels on causal diamonds.

[2] Here, local refers to a description involving a single pixel of a finite area causal diamond in the non-compact space.

[3] In fact, there is no distinction between measurable, continuous and smooth functions at the local level. The different function spaces arise as different limits of the same sequence of finite-dimensional function algebras.

it to be non-commutative, since this gives us a much more efficient, systematic and symmetric way to approximate a classical space. The full Hilbert space of the diamond is the tensor product of the single pixel Hilbert spaces.

The single pixel Hilbert spaces are constructed using the idea that a Lorentzian geometry can be encoded in the orientations in the ambient space-time of all of the pixels (thought of in a naive geometrical way), as well as the holographic areas, of a sufficiently rich set of causal diamonds. At the classical level, the orientation of a pixel is determined by a null ray and a screen element transverse to it. Precisely, this information is contained in a solution of the Cartan–Penrose equation

$$0 = \bar{\psi}\gamma^{\mu}\psi(\gamma_{\mu})_{\alpha}^{\beta}\psi_{\beta},$$

where ψ_{α} is a Dirac spinor. The vector Dirac bilinear is a null vector, and the solution of this equation is a light-front spinor, which determines a transverse plane. The C–P equation is invariant under Lorentz transformations. This local Lorentz invariance is broken to transverse rotations by choosing a gauge in which the light-front spinor S_a^A occupies only the upper components of the Dirac spinor. The classical re-scaling invariance of the C–P equation will be broken to a local Z_2 by our quantization rule. Physically this means that quantum mechanics introduces an area for each pixel, while the classical C–P equation only determines an orientation. The local Z_2 will be identified with $(-1)^F$, which appears to be an exact gauge symmetry of all known consistent string theory models. This choice automatically builds the spin-statistics connection into all holographic space-time models. The local Z_2 also allows us to make a Klein transformation, such that the mutually commuting variables associated with independent pixels become mutually anti-commuting.

With this preface, we can write down our ansatz for the commutation relations:

$$\left[S_a^A(m), S_b^B(n)\right]_+ = \delta_{ab}M^{AB}\delta_{mn}.$$

The labels m, n refer to a basis in the algebra of functions on the holographic screen, so that our variables live in the spinor bundle over the screen. Small latin letters refer to spinors in the d non-compact dimensions of space-time[4]. We assume that these non-compact dimensions have at least an $SO(d-2)$ asymptotic symmetry, and in fact we will assume a larger asymptotic symmetry group later.

The operators M^{AB} live in the space of forms at a point on the internal manifold. We will take the dimension of this manifold to be $11-d$, anticipating a connection to supergravity. It is natural to suppose that the operators corresponding to forms of a given degree, p, can be written in terms of sums over more primitive operators, corresponding (in the geometric limit) to independent p-cycles on the internal manifold, so that all of these operators have the interpretation of wrapped p-brane charges. These independent p-cycle operators and the pixel operators S_a^A will form a closed finite-dimensional super-algebra. At the moment, all we require of the super-algebra is that it has a finite-dimensional unitary representation.

In a compactification, the pixel label n naturally has a tensor factorization, corresponding to the tensor factorization of the algebra of functions on space-time. The algebra of functions on the compact manifold will always be a finite matrix algebra, so we can enlarge the operators S_a^A and M^{AB} by tensoring in these matrices. Once this is done, the internal geometry, to the extent that it has meaning in the quantum theory, will be encoded in the representation of the super-algebra generated by the brane charges and the pixel operators. String duality has taught us that internal geometries are not absolute concepts in string theory. As we change their moduli (in cases where moduli exist), inequivalent topologies can morph into each other,

[4] It is worth pointing out that in this formalism, de Sitter space is thought of as the maximal causal diamond of a single observer, which is a non-compact space, with boundary.

passing through regions where no geometrical description exists. It is only the conserved brane charges which are defined everywhere on moduli space. In a holographic space-time, compact geometry is simply defined in terms of the algebra of brane charges and pixel variables. A classical geometrical interpretation will be valid only in cases which have moduli, and in extreme limits of the moduli space.

One striking feature of this formalism is that the number of 'functions' on the compact space will be finite for any finite area causal diamond. We will see that a convenient way to think of this, for many of the spaces that arise in string theory compactifications, is to use *fuzzy geometry*. The point is that many of these spaces are compact Kahler manifolds, or can be thought of as one-dimensional bundles over a Kahler manifold. Geometric quantization allows us to view the algebras of functions on these spaces as limits of sequences of finite matrix algebras. We will discuss some of the striking conceptual consequences of this observation in section 3.

2.1. The Hilbert spaces of single observers and their intersection

In Lorentzian space-time with no closed time-like curves, an observer is modeled by a time-like world line whose tangent vector is everywhere future directed. In quantum mechanics, the term observer refers to a large system whose internal dynamics are well approximated by a cut-off quantum field theory in a volume large compared to the cut-off scale. The pointer variables of this observer are averages of local fields over large volumes. Measurements consist of dynamical entanglement of microscopic variables with the large ensembles of states corresponding to fixed values of the pointer variables. Such measurements destroy quantum coherence up to small corrections of order e^{-V} where V is the volume of the pointer in cut-off units.

In the real world, any such measuring device will have a mass and travel on a time-like world line. In holographic space-time, we model such a world line as a nested sequence of causal diamonds whose tips have larger and larger time-like separation along the world line. For small enough time-like separation, the holographic screens of these diamonds will have finite area. In space-times with an asymptotic causal structure like that of Minkowski or de Sitter space, the screen area will be finite for all finite time-like separation[5], while in space-times like AdS, the area goes to infinity at finite time. In all space-times with both non-singular past and future[6], it is convenient to view the nested diamonds of a single observer to be centered about a single point on the observer's trajectory. In Big Bang space-times, we instead take a sequence of diamonds whose past tips all lie on the Big Bang singularity. The latter rule will allow us to incorporate the notion of *particle horizon* into our formalism.

The quantum translation of these statements, whose validity we assert even in regimes where no sensible geometrical picture exists, is a sequence of Hilbert spaces $\mathcal{H}(t, \mathbf{x})$, such that $\mathcal{H}(t, \mathbf{x}) = \mathcal{P} \otimes \mathcal{H}(t-1, \mathbf{x})$. The label $\mathbf{x}$ indicates a position on a ($d_S = d-1$)-dimensional spatial lattice. This lattice determines the topology (but not the geometry) of a space-like slice $\mathbf{S}$ *in the non-compact dimensions*. As noted above, the topology of the compact dimensions is not an invariant concept. The compact invariants are the brane charges, which appear in the pixel super-algebra. They take on topological meaning only when there are extreme regions of moduli space. $\mathcal{P}$ is the representation space of the pixel super-algebra. In a cosmological situation, the change of properties of the compact space with cosmological time would be encoded in t dependence of the super-algebra.

[5] In dS space the area remains finite for any time-like separation.

[6] We will use the phrase *scattering space-times* to describe space times whose asymptotic past and future are both non-singular.

The space-like slice **S** should be thought of as the cosmological initial surface for Big Bang space-times. For scattering space times each observer is, loosely speaking, described by a sequence of causal diamonds centered on some point. The 'initial' space-like slice is the one which goes through the central points of all the observers on the lattice. For such space-times, we have, instead of Hamiltonian dynamics, better and better approximations to the scattering matrix, in terms of matrices that compute outgoing from incoming data, in finite causal diamonds.

It should be noted that in situations where a geometrical description is accurate, the integer variable t is a monotonic measure of the proper time τ traversed between the past and future tips of the diamond represented by $\mathcal{H}(t, \mathbf{x})$, but they are not linearly related. In situations where the internal geometry is unchanging and the external geometry is weakly curved, each increment in t represents an equal increase in the area of the diamond, so $t \propto \tau^{d-2}$. This implies that as causal diamonds get bigger, the proper time is discretized in smaller and smaller units. Roughly speaking, the smallest proper time interval measurable in a large causal diamond is inversely proportional to the energy of a black hole whose horizon area is equal to the area of the holographic screen.

The rest of the kinematical specification of a holographic quantum geometry consists of a set of overlap rules. We specify that

$$\mathcal{H}(t, \mathbf{x}) = \mathcal{O}(t, \mathbf{x}, \mathbf{y}) \otimes \mathcal{N}(t, \mathbf{x}, \mathbf{y}),$$
$$\mathcal{H}(t, \mathbf{y}) = \mathcal{O}(t, \mathbf{x}, \mathbf{y}) \otimes \mathcal{N}(t, \mathbf{y}, \mathbf{x}).$$

Note that the overlap Hilbert space $\mathcal{O}$ is the same for **x** and **y**, but the Hilbert spaces $\mathcal{N}$ may be different. For nearest neighbor points on the lattice, we insist that

$$\mathcal{N}(t, \mathbf{x}, \mathbf{y}) = \mathcal{N}(t, \mathbf{y}, \mathbf{x}) = \mathcal{P},$$

for all t. Geometrically, this is the requirement that the trajectories of nearest neighbor observers always share all but one pixel's worth of information. The dimension of the overlap Hilbert space is required to be a monotonically non-increasing function of the minimal number of lattice steps between **x** and **y**. The rest of its specification is part of the dynamics of the system.

To discuss the dynamics, we must make some specification of the space-time asymptotics. We distinguish three cases:

- Space-times, like dS space, in which there is a maximal area causal diamond.
- Space-times, like Minkowski space, where the area of causal diamonds goes to infinity continuously as a function of the proper time in the diamond.
- Space-times, like anti-de Sitter space, in which the area variable t goes to infinity at a finite value of the proper time. Conformal infinity in such space-times is timelike.

We can find examples in the first two categories of both Big Bang and scattering space-times, while in the third category I only know of scattering space-times. The AdS/CFT correspondence suggests that the proper formulation of the quantum dynamics of the third category is in terms of a quantum field theory living on the conformal boundary of space-time. To be more precise, we must insist that the conformal boundary be identical to that of anti-de Sitter space. The rate of approach to the background metric determines whether the QFT is conformally invariant, or is a relevant perturbation of a conformal field theory[7]. The compact dimensions of the bulk space-time are encoded in the target space of the field theory, rather than

[7] It should be noted that the conformal boundary is taken to be a spatial sphere cross time, and the Hamiltonian operator in the far UV of the field theory is the standard conformal generator $K^0 + P^0$.

in the space-time geometry on which the field theory lives. This is analogous to the separation of compact and non-compact dimensions in our general theory of holographic space-time.

Local physics in causal diamonds much smaller than the AdS radius is not easy to disentangle from the field theoretic dynamics on the boundary. There are several proposals to deal with this issue [7], none of them fully satisfactory.

One can also imagine deformations of these field theories in which the exactly marginal and relevant couplings of the boundary theory are allowed to violate the symmetries of the field theory, including time translation invariance. In principle, one could try to model cosmology on field theories with time-dependent couplings, but the time dependence seems quite arbitrary. Furthermore, one would have to specify the initial state of all degrees of freedom in the system, and since the Hamiltonian couples them all at all times, it is not clear why such a 'cosmology' would have particle horizons or sensible local physics.

Instead, Fischler and I proposed [4] to attack the problem of cosmology directly within the holographic formalism. Our mathematically well-defined holographic cosmology is called *the dense black hole fluid* (DBHF). It is defined by the following set of rules:

- Each observer on the spatial lattice is given the same sequence of time-dependent Hamiltonians $H(t)$.[8]
- The operator $H(t)$ is the sum of an operator $H_{in}(t)$ which is built from the pixel operators operating in $\mathcal{P}^t$ and an operator $H_{\text{out}}(t)$ which commutes with all of those operators. This rule builds the concept of particle horizon into the dynamics.
- $H_{in}(t)$ is a perturbation of a bilinear Hamiltonian in the fermionic pixel variables. The bilinear form is chosen independently for each t from the orthogonal ensemble of anti-symmetric $t \times t$ matrices, with a simple t-dependent normalization. For large t, this bilinear Hamiltonian describes a scale-invariant free fermion in 1 + 1 dimensions. The perturbations are required to be irrelevant perturbations of this CFT.
- The overlap rule is $\mathcal{O}(t, \mathbf{x}, \mathbf{y}) = \bigotimes \mathcal{P}^{t-d(\mathbf{x},\mathbf{y})}$, where $d(\mathbf{x}, \mathbf{y})$ is the minimal number of lattice steps between the two lattice points. The Hamiltonians on the overlaps are just $H(t-d(\mathbf{x}, \mathbf{y}))$. As a consequence of the fact that we required the sequence of Hamiltonians to be the same at each lattice point, this rule satisfies all of the complicated consistency conditions.

One can then show that the system has an emergent geometry, which is a spatially flat FRW universe with $p = \rho$. To give some feeling for how this works, the overlap rule gives a formula for the *causal horizon*, the boundary between the set of lattice points with which a given observer has interaction, which is defined by a rule that becomes rotationally invariant at large t, for any regular lattice with the topology of $\mathbf{R}^3$. The size of this causal horizon scales with t, interpreted as the area of the causal diamond reaching back to the Big Bang, as expected for the FRW geometry. The energy density is defined in terms of the Hamiltonian of the system, and also obeys the right scaling law as well as the relation $\sigma = k\rho^{1/2}$ between entropy and energy densities, expected for a $p = \rho$ fluid with extensive entropy. This is also the relation between entropy and energy density for a 'system of horizon filling black holes', which constantly merge to fill the growing horizon. The phrase in quotes is just a heuristic description of the actual mathematics, but it gives rise to the name DBHF that we have given to this system.

The utility of this phrase comes from considering a cosmology consisting of a dilute gas of black holes, with a sufficiently homogeneous distribution. It is clear on the one hand that

[8] The dynamics is discrete and $H(t)$ is just i times the logarithm of the unitary transformation which transforms the system from t to $t - 1$. The parameter t goes from 1 to ∞. All of the operators $H(t)$ operate in the late time Hilbert space $\mathcal{H}(\infty, \mathbf{x})$.

locally the black holes are regions of space-time packed with the maximum allowed entropy, but that on the other hand, this system behaves like an FRW universe with $p = 0$ for a very long time. However, fluctuations in such a universe grow with scale size, and we eventually have to ask what will become of the system. Clearly, fluctuations will lead to larger and larger black holes through collapse and collision. Eventually, perhaps depending on the initial density and fluctuations, we might expect the system to behave like the DBHF.

Quite remarkably, the opposite phase transition from dense to dilute black hole fluid can also occur. This observation is the basis of the realistic cosmology that Fischler and I proposed [4]. It begins with the heuristic idea of a *defect* in the DBHF. Geometrically this is a region of coordinate space, in the flat FRW coordinates defined by the DBHF, in which the dynamics of the system leads to a lower entropy initial configuration. We may imagine that in certain regions we have initial black holes, which are too small to merge with their neighbors in adjacent horizon volumes, as the universe expands. These would decay into particles[9] and this *normal region of space-time* will expand locally like a radiation-dominated FRW universe.

For large t, the system also has a scale invariance, that of the massless $(1+1)$-dimensional fermion field. This can be shown to be the same transformation that implements the conformal isometry $\tau \to c\tau, \mathbf{x} \to c^{\frac{2}{3}}\mathbf{x}$ of the emergent geometry:

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \tau^{2/3}(\mathrm{d}\mathbf{x})^2.$$

The holographic formalism defines a natural time slicing for Big Bang universes, whether spatially homogeneous or not. We simply insist that the integer t, which defines the time slice, refers to the area of the causal diamond associated with the local particle horizon: the intersection of the interior of the backward light-cone of the observer at $\mathbf{x}$ with the interior of the forward light-cone of the Big Bang event at $\mathbf{x}$. In the quantum theory, this is built in to the rule that $\mathcal{H}(t, \mathbf{x}) = \otimes\mathcal{P}^t$. It is easy to see that, in such a slicing, a normal region that *expands freely* has spatial volume growing more rapidly than that of the DBHF. Thus, the spatial volume fraction of normal region relative to DBHF grows with time. As a consequence, at some physical size M for the cosmological horizon, the normal patch of the universe looks like a radiation-filled universe interspersed with patches of DBHF. The latter behave like black holes: local regions of maximum entropy, with all of their degrees of freedom in equilibrium. Thus, we have achieved a transition between the DBHF and a dilute black hole gas. If the dilute black hole gas is approximately homogeneous, this system will quickly begin to evolve like a $p = 0$ FRW universe, but well-known gravitational instabilities will bring it back to the DBHF very quickly, unless the inhomogeneities are very small. *Thus, holographic cosmology can explain the low entropy of the initial conditions for the normal part of the universe. One would hope to eventually prove that any higher entropy of normal degrees of freedom would lead to re-collapse to the DBHF*. A period of inflation can help to avoid this instability, if the theory contains a low energy effective inflaton field. The holographic framework provides the explanation for why initial configurations of this field *must* be approximately homogeneous: again to avoid re-collapse into the DBHF fluid. The reader should note carefully that homogeneity, isotropy and flatness are, in this formalism, all derived for generic initial conditions, without inflation. Indeed, I do not believe that the convention *inflationary* explanation of these conditions really explains anything, because it makes drastic assumptions about the initial state of those degrees of freedom that cannot be approximately described by local field theory in the initial inflationary patch. The real test of the explanation of low entropy initial conditions from the principle of avoiding re-collapse to the DBHF would be a first principle calculation of the amplitude of primordial density fluctuations. This does not yet exist.

9 The right way to think about particles in the holographic space-time formalism will be adumbrated below.

The picture sketched in the previous paragraph depended on the dynamical assumption that the normal region could expand as if the surrounding DBHF was not there. A rigorous investigation of this requires a fully quantum and holographic description of the normal region and its interactions with the DBHF. However, we can obtain interesting insights by looking at the implications of the Israel junction condition for a spherical region of $p = w\rho$ cosmology embedded into a $p = \rho$ background. We find that, applying the condition that the geometry of the junction be continuous, the coordinate volume of the $p = w\rho$ region shrinks unless $w = \pm 1$. In the de Sitter case, we match the cosmological horizon, which is a marginally trapped surface, to the horizon of a black hole geometry of equal area in the $p = \rho$ background. This implies that there is a stable equilibrium between an asymptotically de Sitter normal region and the DBHF, which does not exist for any other asymptotic FRW geometry.

I view this as a prediction of a positive cosmological constant, whose value is determined by initial conditions. One can start with many finite defects in the DBHF, each one involving some finite number of the pixel variables[10], which would evolve to a *lonely multiverse* of isolated asymptotically dS universes. I call it lonely, because there is no reason for these universes to have any effect on each other. It would be an amusing problem in GR to determine the properties of a solution with two trapped surfaces in a space-time asymptotic to the flat $p = \rho$ universe. Do they attract, repel, collide? Whatever the answer however, one can surely choose the initial distribution of universes in such a way that the collision takes place only long after each universe has reached its asymptotic dS regime. Thus, depending on unknown initial conditions we could always arrange that there is no observable effect of collisions even if they occur. We might as well insist that no such collisions occur.

Returning to the Israel argument for stability, we note that the entropy associated with the dS space coincides with that excised from the $p = \rho$ universe, so the thermodynamic argument for stability mirrors the geometrical one. Another way of putting this is that the de Sitter vacuum is a state which maximizes the entropy in a given causal diamond. In the DBHF, the degrees of freedom of this diamond would mix with the other degrees of freedom in the universe, but by modifying the metric around the de Sitter bubble to be that of the '$p = \rho$ Schwarzschild solution' we find a stable solution of general relativity, and thus, local thermodynamic stability. If we treat fluctuations around this solution by the methods of quantum field theory, we would find a Hawking instability, but our explicit model of the DBHF makes it clear that it has no particle excitations. Indeed, the coarse-grained Friedman equations are derived from a quantum model whose time-dependent Hamiltonian is constantly moving around the state vector in Hilbert space, rather than a system with a unique time-independent ground state. Our claim is that the only stable 'excitations' of the DBHF are stable black holes with de Sitter interiors.

The Israel condition argument also implies that the initial normal region must have a complex shape. If it were spherical, it would be invaded by the DBHF before it reached its asymptotic dS limit. To be more precise, we have two choices:

- We can assume that the initial normal region is spherical, but takes up a much larger coordinate volume (more points on the lattice) than our current horizon volume does, so that even though the coordinate volume shrinks in response to the pressure of the external $p = \rho$ region, our full horizon volume survives until de Sitter expansion begins, or
- We assume a non-spherical shape, determined by maximizing the initial fraction of the coordinate volume, which is in the DBHF fluid phase, *subject to the constraint that the phase transition to a more normal universe occurs, and that the normal phase remains*

[10] Recall however that we do not yet have a description of the defects in terms of pixel variables.

stable until de Sitter expansion takes over. We have argued that the de Sitter phase of expansion of the normal universe is stable against re-collapse into the DBHF.

There is more entropy in the second kind of initial condition, and indeed the first kind is simply a low entropy example of the second. It is attractive to assume that the initial state of the universe is as generic as possible, since we then feel no compulsion to explain its properties. We have seen that the most generic initial conditions lead to the DBHF, a universe in which nothing ever really happens. Within the bounds of our ignorance about the detailed mathematical formulation about the DBHF-defect model of the universe, it seems reasonable to characterize it as the most generic initial state that can lead to complex evolutionary behavior in the future.

To summarize, holographic cosmology predicts that the normal region of the universe must asymptotically approach de Sitter space, with a cosmological constant determined by the cosmological initial conditions. Λ is determined by the number of degrees of freedom that were initially out of equilibrium with the background DBHF. In other words, holographic cosmology is automatically a *multiverse* theory. Separated small defects in the DBHF evolve into normal universes with a variety of values of the cosmological constant. Our own universe can thus be subject to environmental selection effects. There is however an *a priori* preference for the largest value of the c.c. that can be compatible with the anthropic constraints, since that represents the maximum entropy initial condition for the universe. The answer to the question of whether other parameters in the Lagrangian describing low energy physics are determined environmentally depends on the degree of uniqueness of the quantum theory of de Sitter space, a subject to which we now turn.

3. The quantum theory of stable de Sitter space

3.1. The two Hamiltonians of Willem de Sitter

We begin by recalling some semi-classical properties of de Sitter space. We will work in four dimensions, inside a single horizon volume, which may be covered by a static metric

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2.$$

For the dS 'vacuum', $f(r) = (1 - (r/R)^2)$, while for the Schwarzschild–dS black hole, $f(r) = \left(1 - \frac{2M}{rM_P^2} - (r/R)^2\right)$. Gibbons and Hawking [14] argued that both of these metrics represented thermodynamic ensembles of states. In particular, the vacuum ensemble is canonical, with *unique* temperature $T^{-1} = 2\pi R$ and entropy $S = \pi(RM_P)^2$. The black hole metric has two horizons, each with its own entropy and temperature. For $M \ll M_P^2 R$, the inner horizon is approximately that of a Minkowski black hole, while the outer horizon is approximately that of the vacuum ensemble. It is important that the combined entropy is *always* less than that of empty dS space. For small M the entropy deficit is $2\pi RM$.

The hypothesis that the entropy of dS space is just the logarithm of the dimension of the Hilbert space in its quantum theory [5] leads to a natural interpretation of these facts. Remarkably, the explanation invokes *two* different Hamiltonians. The first, which we denote by H, is an operator with spectrum bounded by cT, with c a constant that has not yet been determined. It is likely that it will turn out to be a 'random' operator, in that all properties of dS space which are *in principle* amenable to measurement will depend only on certain gross properties of H. The first is that H has a chaotic spectrum, so that a generic initial state cycles through the entire Hilbert space under H evolution. This statement is meaningful only for a Hilbert space of very large dimension, but that will certainly be the case for the Hilbert space representing our own universe. We will see that the quantum theory of dS space really only makes sense when its entropy is large. The vacuum ensemble of Gibbons and Hawking will be identified with the ensemble of all states of a Hilbert space of dimension $e^{\pi(RM_P)^2}$, evolving under the Hamiltonian H.

Given this identification, we can immediately understand another aspect of semi-classical de Sitter physics. The Coleman–DeLucia [15] instanton for transitions between two dS vacua is a compact Euclidean manifold with negative Euclidean action. It defines *two* transition probabilities upon subtraction of the Euclidean actions of the two dS spaces. These describe inverse processes and the probabilities are related by

$$P_{12} = P_{21}\, e^{S_1 - S_2}.$$

This is the principle of detailed balance for a system in equilibrium at infinite temperature, which is precisely the interpretation of the dS vacuum state we gave in the previous paragraph.

The Hamiltonian H is certainly not the Hamiltonian whose eigenvalues are particle masses in the real world, and the vacuum ensemble is an infinite temperature ensemble for H. This indicates the need for another Hamiltonian, P_0, to describe local physics that is approximately Poincaré invariant in the large RM_P limit. Indeed, the manner in which the cosmological causal diamond in dS space approaches Minkowski space indicates the existence of two Hamiltonians. Near the cosmological horizon, the static metric approaches

$$ds^2 = R^2(-du\, dv + d\Omega^2),$$

where the horizon is the surface $v = 0$. The metric of asymptotically flat space near conformal infinity is

$$ds^2 = \frac{1}{v^2}(-du\, dv + d\Omega^2).$$

The relation between the two in the $R \to \infty$ limit is clear. The Lorentz group is realized as the conformal group of the two sphere, and the translation generators are

$$P_\mu \propto (1, \Omega)\partial_u.$$

By contrast, the static Hamiltonian acts on the dS metric as

$$H \propto (u\partial_u - v\partial_v),$$

so that

$$[H, P_\mu] \propto P_\mu.$$

Global dS space does not have an infinite null or time-like boundary, so it is not clear from the canonical formalism of gravity how one should interpret its isometries. One can argue that they are all gauge generators, which should be set to zero. However, such a formal argument would apply to any finite causal diamond in the holographic formalism. Instead, one should look at this formalism as working in a fixed physical gauge, defined by some set of physical measuring devices.

My current understanding of the quantum theory of measurement relies on (cut-off) quantum field theory. A measurement consists of a dynamical entanglement of some microstate with the ensemble of states corresponding to a fixed position of a pointer variable. Pointer variables are averages of local fields over volumes large compared to the cutoff. Tunneling between states corresponding to different pointer positions is suppressed by $e^{-V_{\text{pointer}}}$, with the volume measured in cut-off units. Thus, I believe that the only kind of states for which we have a reliable measurement theory are those which are localizable. They are either described by bulk quantum field theory, or consist of black holes small enough to have their states measured by devices that obey quantum field theory.

In dS space, all such states are evanescent and decay eventually to the dS vacuum. Our proposed model for these facts consists of the dS Hamiltonian H of a few paragraphs back and an operator P_0 satisfying

$$[H, P_0] = M_P^2 g\left(\frac{P_0}{RM_P^2}\right),$$

(*cf the commutator of generators on the cosmological horizon*) where $g(x)$ is a smooth function which is $o(x)$ for small x. The spectrum of P_0 runs from 0 to the (Nariai) mass of the maximal black hole in dS space. For eigenvalues of P_0 small on the Nariai scale, the entropy deficit of the eigenspace, relative to the full dS Hilbert space, must be $2\pi RP_0$. This implies that the infinite temperature ensemble for H is the dS temperature ensemble for P_0 as required by the match to semi-classical physics. Note that this relation between entropy deficit and eigenvalue is *precisely* what we observed above for black holes, if we identify the black hole mass with an approximate eigenvalue of P_0. We will provide a somewhat more refined, but still crude, model of P_0 in the third subsection of this chapter.

Returning to what we said about CDL transition probabilities, the quantum interpretation of the CDL transition refers to the Hamiltonian H, rather than the emergent Hamiltonian P_0, whose spectrum consists of states with lifetimes short compared to the CDL transition time scale. While we are on the subject of CDL instantons, it is worthwhile pointing out the distinction between stable and unstable dS spaces, which appears in the CDL formalism. If the potential contains a zero c.c. minimum or an asymptotic region where the energy density is zero, then dS space is unstable to decay to the zero c.c. region. There is no inverse transition, and this is interpreted by saying that the zero c.c. configuration represents a system with an infinite number of states. If there is no zero-energy point on the potential, even at infinity in field space, then the space of potentials divides into two classes [8]. The distinction is based on the behavior of tunneling amplitudes in the limit that the lowest dS minimum is shifted to zero. If the resulting Minkowski space has a positive energy theorem, so that it is stable, then all 'decays' of the lowest dS minimum including those to negative c.c. Big crunches behave like $e^{-\pi(RM_P)^2}$ for large dS radius, and can be interpreted as improbable transitions to low entropy states of a finite system (the lowest dS vacuum). If the decay proceeds even in the Minkowski limit, then the dS space is unstable, and it is not clear whether the system has a sensible quantum mechanical interpretation.

My claim that the negative c.c. crunches, must, in some cases, transition back to empty de Sitter space, has caused some raised eyebrows. What I want to emphasize is that this *must* be the case if one assumes that the quantum system being described has a number of states bounded by the exponential of one quarter the area of the maximal sized causal diamond in the space-time. The reverse transitions then follow from unitarity. Reference [9] provides evidence that the singular CDL crunches indeed correspond to subsystems with microscopically small entropy.

3.2. The variables of dS quantum mechanics

In accord with our general formalism, the variables of a quantum dS space satisfy the anti-commutation relations

$$\left[(\psi^a)_i^A, (\psi^{\dagger b})_B^j\right]_+ = \delta_i^j \delta_B^A M^{ab}.$$

Here, a, b are 8-component spinor indices and the super-algebra of M and ψ specifies the geometry of compactified dimensions. The indices i and A run from 1 to N and 1 to $N + 1$, respectively, and the $SU(2)$ rotation symmetry of the cosmological horizon acts on these like a section of the spinor bundle over the fuzzy 2-sphere. The entropy of this system is

$N(N+1)\ln D$, where D is the dimension of the representation of the compactification super-algebra. For large N, this is identified with $\pi(RM_P)^2$, with M_P being the four-dimensional Planck scale. Thus $N \sim RM_P$.

I have not yet worked out the theory of fuzzy compactifications, so I will follow [10] and drop the internal spinor indices and the brane charges M^{ab}. This leads to a Hilbert space containing only chiral multiplets of minimal four-dimensional SUSY, and no graviton. Nonetheless, we will be able to see how particles, supersymmetry and black holes arise in the large N limit.

Note by the way that we only try to construct dS space of dimension 4, where the holographic screen is a two sphere. This is motivated by supergravity. For large dS radius, the quantum theory should contain states which are well described by supergravity, and for self-consistency, the supergravity Lagrangian should have dS solutions. The only examples I know have at most four supercharges, which implies four or fewer dimensions. The interesting physics in dS space is the almost Poincaré invariant physics of localizable states. In fewer than four dimensions there is no sensible notion of an S-matrix in the presence of supergravity.

The clue to the proper description of particle states in dS space comes from a simple exercise first done in [1]. We ask how to maximize the entropy described by quantum field theory in a single horizon volume. In field theory, one maximizes entropy by going to high energy. In a region of size R the entropy of a field theory will be of order $(MR)^3$, where M is the ultra-violet cut-off. The mass in this region is of order $M(MR)^3$. The Schwarzschild radius M^4R^3/M_P^2 of this mass must be less than R. Otherwise we are talking about black hole states, for which the field theory description is incorrect. This leads to a bound $M < (M_P/R)^{-1/2}$. Of course, this cannot be an absolute bound on the momentum of any particle. Rather, it is a bound on the momentum of the particle states of maximum entropy, which can exist in dS space. Fewer particles of higher momentum would also evade the black hole bound. Field theory does not provide a concise description of how to describe such a restriction on the allowed-particle states. We will see that the holographic description accomplishes this in an elegant manner.

The forgoing argument shows us how the formalism of quantum field theory in curved space-time, in which there appear to be an infinite number of copies of the degrees of freedom in a single horizon volume at large values of the global time, might emerge as a limit of the quantum dS formalism. The field theoretic entropy in a horizon volume is of order $(RM_P)^{3/2}$. Consequently, the Gibbons–Hawking entropy allows us to have of order $(RM_P)^{1/2}$ independent copies of these degrees of freedom. The field theory prediction is recovered in the limit $(RM_P) \to \infty$.

The holographic description of these particle states is obtained via the block decomposition of the fermionic matrix:

$$\psi_i^A = \begin{pmatrix} 1 & 2 & 3 & \dots & K \\ K & 1 & 2 & \dots & K-1 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 2 & 3 & 4\dots 1 & & \end{pmatrix}.$$

Each diagonal labeled by the same integer represents the degrees of freedom in a single horizon volume, while each block within this diagonal represents the degrees of freedom of a single particle in this horizon volume. In order to have all horizons equivalent, we must have $K \sim N^{1/2}$. If, following the suggestion of matrix theory [3] we now identify the radial component of momentum of a particle, in units of the dS temperature with the trace of the block size, then our typical momentum is of order $N^{-\frac{1}{2}}$ in Planck units, which is the same as the momentum cutoff in our heuristic field theory argument. Larger values of momentum can be obtained by lowering the entropy.

To understand this we note that in the large N limit, the operators in each block of the single horizon volume diagonal converge to

$$\psi\sqrt{p}\delta(\Omega - \Omega_0),$$

where ψ is a single fermionic annihilation operator, and Ω_0 runs over the two sphere. The block sizes all go to infinity with fixed ratio, and physics becomes invariant under re-scaling of the block sizes: this is Lorentz boost invariance under boosts in the Ω_0 direction[11]. As promised, the variables are sections of the spinor bundle over the two sphere[12]. The positive real number p is the re-scaled trace of the unit matrix in a single block. It is interpreted as the overall scale $p(1, \Omega_0)$ of the light-like momentum of a massless superparticle, which exits the holographic screen in direction Ω_0. The rotation generators acting on the rows and columns of the single $K \times K + 1$ block identify the representation of SUSY as the chiral supermultiplet. The SUSY generators are

$$Q_\alpha = \psi q_\alpha(\Omega_0),$$

and their complex conjugates. $q_\alpha(\Omega_0)$ is one of the conformal Killing spinors of the two sphere, which transform like a left-handed Weyl spinor under the $SO(1, 3)$ conformal group. The latter is, as usual, identified with the Lorentz group, and we worke in a basis where the single particle momentum is diagonal. Note that the limit from dS space picks out a special Lorentz frame, the one in which a particular static dS observer is at rest.

For finite N, the trace of a block is interpreted as the overall scale of momentum, in units of $1/R$. Thus, for the maximally entropic single horizon particle configurations where $K \sim \sqrt{N}$, the momentum is of order $R^{-\frac{1}{2}}M_P^{1/2} \sim \Lambda^{1/4}$. We can obtain higher momentum by recognizing that particles are defined here as in experimental physics: by their imprints on the detector (the holographic screen). Thus, B blocks which exit the same pixel (have the same angular momentum wavefunction) will be interpreted as a single particle with momentum $B\Lambda^{1/4}$. Since B can be as large as $N \sim (RM_P)$, the momentum can be as large as $10^{30}M_P$. The physical origin of this bound is a bit obscure. Any particle in dS space will scatter off the Gibbons–Hawking radiation, which would create a black hole for high enough particle momentum. However, a typical GH quantum has energy of order the dS temperature, so the center of mass energy in the collision would be of order $\sqrt{ET}$ and it would seem that the threshold for black hole production is $E \sim M_P(RM_P)$, rather than $E \sim (RM_P)^{\frac{1}{2}}M_P$. We will see in the following section that black hole states in a given horizon volume indeed borrow particle degrees of freedom from other horizons and that black holes are like particles with very high momentum, at least insofar as their count of degrees of freedom is concerned. They differ from particles in that they generically do not have multi-black hole states related by permutation symmetry.

For the time being, I will leave this small puzzle about the maximum momentum unresolved and proceed to the discussion of SUSY breaking. We have seen that in the large N limit, the full super-Poincaré algebra emerges. For generic particle states, with momentum of order $\frac{\sqrt{N}}{R}$, the corrections to the super-Poincaré algebra would be of order $N^{-\frac{1}{2}} \sim (RM_P)^{-\frac{1}{2}} \sim \left(\frac{\Lambda}{M_P^4}\right)^{1/4}$. The corrections to the commutator of Q_α and P_0 should be of this order, measured in Planck units. Recalling that in the super-Higgs mechanism, the superpartner of any state is a state with an additional massive gravitino, we get the prediction

[11] I should emphasize that throughout this paper we deal with kinematics. The requirement that the dynamical S-matrix be invariant under Lorentz transformations is a strong constraint on the dynamics.

[12] To be more precise: they are elements of the dual space to the space of measurable spinor sections.

$$m_{3/2} = \kappa \Lambda^{1/4}.$$

Interestingly, this coincides with a heuristic estimate [12], which we review in the appendix, based on a model of gravitino interactions with a random system spread over the horizon with a uniform density of states.

The estimate of the mass scale of standard model superpartners, which follows from combining this equation with gravitational effective field theory, is $M_S = \sqrt{\frac{\kappa \Lambda^{1/4} M_P}{(8\pi)^{1/2}}} = \sqrt{2\sqrt{10}\kappa}$ TeV. This is of course as low as it could possibly be and still be consistent with current experimental bounds. The precise estimate depends on the unknown constant κ.

The strategy I have adopted for pursuing the phenomenological consequences of these ideas is based on the fact that holographic cosmology implies that N is a cosmological initial condition, and is therefore a freely variable input parameter. In the large N limit, SUSY breaking is a very low energy phenomenon and, apart from the question of what fixes the cosmological constant, it should be understood in low energy effective field theory. In particular, the $N = \infty$ theory should be supersymmetric and R symmetric (to explain the vanishing of Λ in the limit). The R symmetry will be discrete, in accord with general ideas about global continuous symmetries in gravity. For finite N, the Lagrangian will contain R-breaking terms, the nature of which can only be computed from the full quantum theory of dS space. These must induce a SUSY violating vacuum state. The *size* of the R-breaking interactions is determined by the requirement that the gravitino mass, a quantity which can be calculated reliably in local field theory, be of order $\Lambda^{1/4}$. A constant term in the superpotential is added to make sure that the c.c. is of order Λ. We do not worry about the fine tuning required for this parameter, because it implements a property that we know to be imposed by the full quantum theory. The most phenomenologically successful implementation of this strategy is called the Pentagon model and is reviewed in [11].

Returning to the underlying quantum theory, we note that different blocks of the same size are related by gauge transformations: change of basis in the ψ_i^A index space. It is important to note that whereas the i and A indices are chosen to transform under the rotations of the cosmological horizon, this is not the case for the emergent Lorentz group in the $N \to \infty$ limit. Instead, rotations act on the indices within individual blocks, whereas the transformations that exchange blocks are viewed as gauge equivalences. We have seen above that the Hamiltonian H is distinct from the emergent Hamiltonian P_0, which acts on localizable states in a single horizon volume and becomes a generator of the Poincaré group in the large N limit. Now we see that the generators of $SO(3)$ rotations in dS space are not the same as their Poincaré limits.

It is amusing to note that the unitary transformations which exchange diagonals are also viewed as gauge transformations in this formalism. These are discrete analogs of elements of the $SO(1, 4)$ dS group. It has been suggested [17] that the quantum theory of dS space might be invariant under a q-deformed version of the dS group, in order to be consistent with a finite-dimensional Hilbert space. Perhaps these discrete gauge transformations should be thought of in this light.

At any rate, it has become abundantly clear that the local physics in dS space has little to do with generators which act globally on the space. Instead it is encoded in the emergent super-Poincaré group of the $R \to \infty$ limit.

4. Black holes in dS space

The metric of a Schwarzschild–de Sitter black hole is

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2,$$

where

$$f(r) = 1 - R_s/r - (r/R)^2.$$

The horizons are at the positive real zeros of $f(r)$ and satisfy

$$R_+R_-(R_+ + R_-) = R_sR^2$$

and

$$R_+R_- + R_+^2 + R_-^2 = R^2,$$

where $R_s = 2M/M_P^2$ is the Schwarzschild radius of a black hole of mass M in asymptotically flat space-time. In accord with the notation, we insist that $R_+ > R_-$. The second of these equations shows that the combined entropy of black hole and cosmological horizons is less than that of empty dS space. This leads us to the conclusion that localized excitations all have an entropy deficit: they are not typical states in the dS vacuum ensemble (which we recall is the infinite temperature ensemble for the Hamiltonian H). The first equation tells us that the Schwarzschild radius, and thus the mass, is determined by the entropy deficit. For $R_s \ll R$ we have $\Delta S = (M/2\pi R)$. Assuming that this relation continues to hold for less massive localized excitations, which are not black holes, we derive the Gibbons–Hawking thermal spectrum. Finally, we note the fact that R_- cannot be increased indefinitely: its maximum occurs when $R_+ = R_- = R/\sqrt{3}$.

It is easy to model these properties in terms of our fermionic oscillators. Define, in Planck units,

$$\pi R^2 \approx \ln 2N^2, \qquad \pi R_\pm^2 \approx \ln 2N_\pm^2.$$

More precisely, fix an integer N_- and define N_+ to be the closest integer approximation to the solution of

$$N_+N_- + N_+^2 + N_-^2 = N^2.$$

Define the ensemble of black hole states to be those satisfying

$$\psi_i^A|BH\rangle = 0, \qquad i = 1, \ldots, N_-, \quad A = 1, \ldots, N_+.$$

The basis in which this is true is chosen arbitrarily. By analogy with our discussion of particle states, we may think of this as a choice of the horizon volume in which the black hole sits.

The entropy deficit of this ensemble, relative to the maximally uncertain density matrix, is $N_+N_-\ln 2$, which is, for large N_-, what we expect from a Schwarzschild–dS black hole, given our identification of horizon radii with $N_\pm$. We can then invent a Poincaré Hamiltonian P_0 such that the black hole ensemble consists mostly of states whose eigenvalue satisfies the classical relation between mass and entropy. To do this we note that the statistical expectation value of the fermion number operator,

$$\mathbf{N} \equiv (\psi^\dagger)^j_B \psi^B_j,$$

is

$$\langle\langle \mathbf{N} \rangle\rangle = \tfrac{1}{2}(N - N_+N_-),$$

and its fluctuations are of order $1/N$ for large N.

Thus, if

$$P_0 \equiv \sqrt{\frac{\ln 2}{2\pi}} M_P \left(1 - \frac{2\mathbf{N}}{N^2}\right) \sqrt{N^2 - \mathbf{N}},$$

and we make the above identifications of integers with radii, then

$$\langle\langle P_0 \rangle\rangle = M.$$

This equation should be understood as what I have elsewhere [6] called the *asymptotic darkness* approximation: black holes are, in this approximation, degenerate eigenstates of the high energy limit of the Hamiltonian. The explicit construction adds a new wrinkle: even in this approximation the states are non-degenerate and the black hole energy is a statistical average. Improvements to the asymptotic darkness approximation will lift the degeneracy of states of equal $\mathbf{N}$ and replace it by a random, closely spaced set of levels with density $2^{-N_-^2}$. They will also allow black holes to decay into particle states[13], and it is likely that low entropy members of the approximate black hole ensemble with $\mathbf{N}$ far from $\langle\langle\mathbf{N}\rangle\rangle R$ will be particles rather than black holes.

The last few sentences were fantasies of hypothetical future work. What we have accomplished in this section is a construction of states and a Hamiltonian with the qualitative features of semi-classical black holes, and we have constructed them out of the same variables that we used above to construct particle states. It is reasonably clear that very large black holes, near the Nariai maximum, will not admit particle excitations, and that groups of particles with large momenta will naturally merge into black holes. The details (in which, famously, the devil resides) remain to be worked out.

5. Conclusions

The formalism of holographic space-time is a fully quantum mechanical system of axioms, in which space-time geometry is an emergent property of a class of large quantum systems. The causal structure of the space-time is fixed, but is determined in terms of possible solutions to an infinite set of dynamical consistency conditions. So far, the only known solution of these equations is the DBHF cosmology of [10]. This is a mathematically well-defined model and forms the starting point for a more realistic cosmology based on the idea of normal defects in the DBHF. Using the Israel junction condition and simple scaling arguments, holographic cosmology provides the first complete theory of the initial conditions of the universe. In particular, it provides a rationale for the low initial entropy one must assume in standard cosmology. The system undergoes a phase transition to a dilute black hole gas at a certain scale of energy density, well below the Planck scale. Prior to this transition, the formalism of quantum field theory is not a good approximation to the physics. Just after the transition, the system is well modeled by a gas of black holes whose size is a bit smaller than the particle horizon. If the gas is relatively homogeneous, this gas will expand freely and the black holes will evaporate. If not, it will re-collapse into the DBHF.

The DBHF-defect model thus derives the homogeneity, isotropy, flatness, and low initial entropy of the universe without recourse to inflation. It cannot however account for the long range correlations in the CMB data, without a small amount of inflation (perhaps 20 e-folds), but it does provide the rationale for the homogeneous initial conditions assumed in most inflationary models[14].

Finally, the Israel junction condition implies that any region of space-time that has a normal equation of state in the asymptotic future must evolve to de Sitter space. One dS horizon volume is embedded in the $p = \rho$ DBHF background as a marginally trapped surface. The c.c. of this asymptotically dS space time is determined by cosmological initial conditions. It counts the number of degrees of freedom that have escaped falling into the equilibrated DBHF

[13] The phenomenology of Hawking decay means that the width of these states is much larger than the level density.

[14] I believe that the standard claim that inflation automatically explains this (the universe is a free lunch) is based on a highly un-natural assumption that most of the degrees of freedom of the current universe, which *cannot* be modeled by quantum field theory in the initial inflationary patch, are in the ground state of some adiabatic Hamiltonian. This puts in rather than derives the very special nature of the initial state.

fluid. We are led to the concept of a *lonely multiverse*, a universe composed of a distribution of normal cosmologies, all asymptotically future dS, embedded as marginally trapped surfaces in a $p = \rho$ background. The *a priori* measure on the cosmological constant favors large values of Λ, but the initial amplitude of density fluctuations is bounded[15]. Anthropic considerations [18] favor smaller values of Λ, so we might imagine that the observed value is a compromise between these two criteria[16].

One may ask whether other parameters of low energy physics have a similar random distribution[17]. In the context of holographic space-time, this is the question of how many different theories of stable dS space we can construct. It should be noted that whether or not the string landscape of meta-stable dS spaces exists, none of them can be the same as the models which one will construct by holographic methods. The latter have, by construction, a finite number of quantum states, whereas the former, by virtue of their decays into zero c.c. regions of moduli space, seem to require an infinite number.

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Appendix

A.1. The sound of one hand waving at the horizon

In the text, I argued for the relation $m_{3/2} = \kappa\Lambda^{1/4}$ because $N^{1/2} \sim (M_P/\Lambda^{1/4})$ is the parameter that controls the convergence of the Hilbert space of single particle states to its semi-classical limit. $N^{1/2}$ is the cutoff on single particle angular momenta. In this appendix, I want to recall another argument for the same scaling.

That argument was based on effective field theory, modified by the interaction of particles with thermal states on the dS horizon. These states are not well described by field theory and are almost exactly degenerate. We recall that the super-Poincaré invariant limiting theory has an R symmetry, which acts on the supercharges like some Z_k phase with $k \geqslant 3$. When this symmetry is unbroken, the low energy effective theory does not have a SUSY violating state. R symmetry is broken by interactions with the horizon, and these induce the SUSY violating state which represents the correct physics in dS space.

Consider the Feynman diagram computation of some R-violating term in the effective Lagrangian near the origin[18]. In order to interact with the horizon degrees of freedom, at least one particle line must propagate out to the horizon, which is a space-like distance $\pi R/2$ away. In order to give an R-violating interaction, that particle must carry R charge and we will assume that the gravitino is the lightest particle with this property: thus we have a suppression

$$\delta\mathcal{L} \sim \mathrm{e}^{-\pi m_{3/2} R}$$

[15] One of the quantitative questions that is so far unanswered is whether this *a priori* bound is close to the observed value of primordial density fluctuations.

[16] The hypothetical connection between SUSY breaking and the c.c., which we discussed, also puts anthropic lower bounds on Λ[11].

[17] With the attendant phenomenological difficulties posed by this hypothesis [16].

[18] Near the horizon, the static observer sees a very high-temperature state in which SUSY is violently broken.

from the two gravitino lines going out to the horizon. The gravitino is absorbed by the horizon via some interaction operator V and then re-emitted by the same operator. Since the horizon states are degenerate we get

$$\delta\mathcal{L} \sim \mathrm{e}^{-\pi m_{3/2} R} \sum_n |\langle g|V|n\rangle|^2.$$

Now we ask: which states of the horizon are likely to give matrix elements of order 1? Like Landau level states, the degenerate horizon states can be localized on the sphere, with a number of states proportional to e^{area} of the localized region.

The hand-waving part of the calculation consists of the following statements:

- Gravitinos, being massive, can only propagate near the null horizon for a proper time of order $m_{3/2}^{-1}$.
- During this proper time, the gravitinos perform a random walk, with Planck step size, on the horizon, thus covering an area $\frac{b}{m_{3/2} M_P}$.

A finite fraction of the states in this area is assumed to have matrix elements of order 1. Thus

$$\delta\mathcal{L} \sim \mathrm{e}^{-\pi m_{3/2} R}\, \mathrm{e}^{\frac{b M_P}{m_{3/2}}}.$$

If $m_{3/2}$ is to have any power law behavior at all when $RM_P \to \infty$, then the positive and negative exponentials must cancel exactly:

$$\pi m_{3/2} R = \frac{b}{m_{3/2} M_P}.$$

Plugging in the relation of R and the cosmological constant we get

$$m_{3/2} = \left(\frac{8b^2}{9\pi}\right)^{1/4} \Lambda^{1/4}.$$

This gives a formula for the unknown constant κ in terms of the unknown constant b.

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